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TORSIONAL MECHANISMS IN DUCTILE BUILDING SYSTEMS

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SUMMARY

It is postulated that in order to estimate torsional effects on the seismic response of ductile building structures, the associated plastic mechanism to be developed in the three-dimensional system should be identified. The proposed approach is very different from that embodied in building codes. Inelastic structures are classified as either torsionally unrestrained or restrained. It is shown that clearly defined mechanisms that are to be mobilized, enable the acceptable system ductility demand to be estimated. This should ensure that the corresponding demands imposed on critical translatory elements of the system do not exceed their established displacement ductility capacity. To this end familiar quantities, such as element yield displacement and stiffness, are redefined. Comparisons are made of the intents of existing codified design approaches and those emphasising the role of imposed inelastic displacements. A simple treatment of the consequences of earthquake-induced inelastic skew displacements is also addressed. The primary aim of the paper is to offer very simple concepts, based on easily identifiable plastic mechanisms, to be utilized in structural design rather than advancement in analyses. Detailed design applications of these concepts are described elsewhere. The approach is an extension of the deterministic philosophy of capacity design, now used in some countries. © 1998 John Wiley & Sons, Ltd.

KEY WORDS: seismic response; buildings; system torsion; ductility; plastic mechanisms

1. INTRODUCTION

In some seismic regions of the world the philosophy of capacity design, developed more than 20 years ago in New Zealand,[1] has been adopted.[2–4] In the capacity design of structures, which could be subjected to significant earthquake-induced inelastic displacements, elements of the primary lateral force-resisting system are chosen and suitably designed and detailed for energy dissipation under severe deformations. All other structural elements are then provided with sufficient strength so that the chosen means of energy dissipation can be maintained.[5] The strategy is thus a deterministic one, whereby, with the choice of an intelligent hierarchy in the strengths of elements, the designer can ensure that a suitable plastic mechanism within the system will be mobilized. One of the aims of the strategy is to ensure that for a given system ductility demand, the corresponding demand on the elements of the mechanism will not be excessive. Depending on the displacement ductility capacity of elements the corresponding system displacement ductility demand can then be estimated.

A typical example in ductile reinforced concrete frames is the choice of weak beams and strong columns, whereby the development of a soft storey is inhibited. Alternately, if a soft storey, with plastic hinges developing at both ends of all columns in that storey, is admitted, then the displacement ductility demand should not exceed the displacement capacity of that storey.

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In terms of inelastic torsional response of a structural system, this strategy does not appear to have been addressed. No code provisions, explicitly allowing for increases in element displacement demands resulting from inelastic storey twist, are known to exist.[6] Therefore, the aim of this presentation is to introduce principles relevant to kinematically admissible torsional plastic mechanisms. These are very simple indeed. Their understanding and exploitation enable the designers to adjust element properties, usually without interference with the functional requirement of a building, so that adverse effects of torsional deformations are minimised or beneficial effects, which do exist, are made use of.

The approach is based on a behaviour oriented study of structural system response to static actions. The aim is to establish clearly defined mechanisms associated with storey translation and storey twist. The study is not concerned with the origin of torsional moments, such as stiffness eccentricities or the dynamically induced inertia of the rotating mass. This approach has also been utilized in the development of specifications for capacity design when applied to ductile reinforced concrete frames.[5,7]

For example, in the process of making columns stronger than beams, allowance is made for the possible influence on elastic columns of higher mode deformation shapes during the fully ductile system response which generates predictable force imputs from the plastified weak beams.

It will be demonstrated that, irrespective of the origin of storey torsional moments, in certain cases the translation of vertical lateral force resisting elements, as affected by a storey twist, can be well controlled. In other cases the identification of torsional mechanism in ductile systems will show that the displacement ductility demand imposed on the system may need to be restricted if the ductility capacity of critical elements, as constructed, is not to be exceeded.

The proposed approach, based on the identification of system behaviour, is design rather than analysis oriented. Therefore, the paper addresses primarily design practitioners rather than researchers involved with the improvement of seismic analysis techniques. The study of plastic mechanism rather than a quantified design strategy is the primary purpose of this paper. Applications in design have been presented elsewhere.[8,9]

2. CONVENTIONAL APPROACH TO TORSIONAL EFFECTS IN BUILDING STRUCTURES

The principles of the torsional response of elastic structural systems are well established. Seismic code provisions utilize these principles the world over.[6] Only for the purpose of illustrating the differences in the approaches to elastic and ductile systems, respectively, the main features of existing codified procedures are summarised with respect to the specific structure shown in Figure 1.

To aid simplicity in the presentation of fundamental principles, an example building has been chosen in which lateral force resistance has been assigned to a set of structural walls only. However, the same procedures apply when elements, such as shown in Figure 1, consist, for example, of reinforced concrete rigid jointed frames. Symbols are defined when first encountered and are also quoted in the list of notations.

The centre of rigidity, also referred to as the centre of stiffness or shear centre, denoted subsequently as CR, is readily established by satisfying the criteria

$$\Sigma x_{ri} k_{yi} = \sum y_{ri} k_{xi} = 0 \tag{1}$$

Element stiffness, for example that of a prismatic reinforced concrete cantilever wall, is usually expressed in the form

$$k_i = \frac{V_{\mathrm{E}i}}{\Delta_i} = \eta \frac{E_{\mathrm{c}} I_{\mathrm{e}}}{h_{\mathrm{w}i}^3} \tag{2}$$

where $V_{\mathrm{E}i}$ is the design base shear assigned to the elastic cantilever, Δ_i is the resulting deflection, for example at the top of the wall with a height $h_{\mathrm{w}i}$, fully restrained at the base and E_{c} is the modulus of elasticity of the concrete and I_{e} is the equivalent second moment of area of the wall section which allows for the reduction of flexural rigidity due to cracking[5,7] and sometimes also for shear deformations. The coefficient η depends only

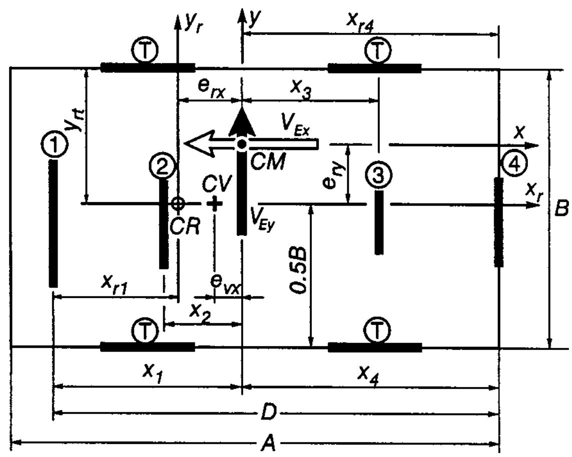

Figure 1. Typical arrangement of elements resisting lateral forces generated by system translation and twist

on the pattern of distributed lateral static forces used in the design. It should be noted that element stiffness, traditionally so defined, is independent of element strength.

Subsequently, the torsional stiffness of the elastic system is found from

$$K_t = \Sigma x_{ri}^2 k_{yi} + \Sigma y_{ri}^2 k_{xi} \tag{3}$$

where x_{ri} and y_{ri} are the element co-ordinates measured from CR. The parameter which defines the ratio of the torsional to translational stiffness of the lateral force carrying elements, with respect to the two principal directions of the framing, x and y, is the radius of gyration of stiffness, defined as

$$r_{kx} = \sqrt{K_t / \Sigma k_{yi}} \tag{4a}$$

$$r_{ky} = \sqrt{K_t / \Sigma k_{xi}} \tag{4b}$$

The distances between CR and the centre of mass, CM, at which the lateral forces in the y and x directions, respectively, are to act, are the stiffness eccentricities, e_{rx} and e_{ry}, shown in Figure 1.

The translational displacement of the elastic system, as a result of the design base shear force, V_E, for example in the y direction, is then

$$\Delta_{Ey} = \frac{V_{Ey}}{\Sigma k_{yi}} \tag{5}$$

The angle of twist resulting from a stiffness eccentricity, e_{rx} is

$$\theta_t = \frac{e_{rx} V_{Ey}}{K_t} \tag{6}$$

Hence, the element force induced by torsion only in the y direction is

$$V_i'' = \frac{x_{ri} k_{yi} e_{rx} V_{Ey}}{K_t} \tag{7}$$

Equations (5) and (6) allow then the displacements of any element due to system translation, Δ_{Ey}, and system twist, θ_t, to be readily determined.

The system translatory stiffness of the structure shown in Figure 1, K_{sy}, for example in the y direction, relating the design base shear, V_{Ey} to the displacement of CM, where the design force acts, may be shown to be

$$K_{sy} = \frac{\sum k_{yi}}{1 + \left(\frac{e_{rx}}{r_{kx}}\right)^2} \tag{8}$$

These quantities may then be used to determine the dynamic properties of the system, such as the periods of vibrations.

Traditional design procedures assign element strength proportionally to element stiffness, defined by equation (2). In this certain assumptions are made with respect to torsional effect. To allow for uncertainties in locating both CR and CM and for resonance effects of translational and torsional vibrations, limits of design stiffness eccentricities, e_d, in the form

$$e_{d1} = \alpha_1 e_r + \beta_1 \tag{9a}$$

$$e_{d2} = \alpha_2 e_r - \beta_2 \tag{9b}$$

are specified. Elements are then designed for the translation and most adverse twist-induced lateral forces. Thereby an increase of the torsional resistance of the system is implied. The estimation of the eccentricity modifiers, α_1, α_2, β_1, β_2, has been extensively examined in the relevant literature.[9,10]

The above summary of the assessment of the torsional response of elastic systems, familiar to most structural designers, is recapitulated here only to serve as a benchmark when the making of comparisons with inelastic seismic response is desirable. In existing provisions in seismic codes[6] only features of elastic response are explicitly addressed in terms of the equations given above.

3. PROPERTIES AFFECTING DUCTILE SEISMIC RESPONSE

It is now well established that for the great majority of structures, particularly those located in regions where, at approaching the ultimate limit state, structural response must rely on significant inelastic displacements. The relevant ability of the structure is conveniently quantified by the system displacement ductility ratio

$$\mu_\Delta = \frac{\Delta_u}{\Delta_y} \tag{10}$$

where Δ_u is the maximum total, or ultimate, displacement of CM and Δ_y is the system yield displacement at the same location. Current force-based seismic design strategies are based on the assumption that the largest displacement ductility demand during a seismic event with a given return period will not exceed the displacement ductility capacity for which the structure has been designed and its construction detailed.

When studying inelastic structural response, it is important to clearly define and quantify, at least with acceptable approximations, those design parameters which characterise inelastic member and system response, such as element and system yield displacements and stiffnesses, where relevant.

3.1. Element yield displacement

As Figure 2 shows, a typical reinforced concrete element, such as a cantilever structural wall, exhibits strongly non-linear moment–curvature relationships. For design purposes the non-linear transition from fully elastic to fully plastic behaviour can be, and has been, conveniently replaced by a bilinear approximations. This approximation enables a reference yield curvature, ϕ_{yi}, or yield displacement, Δ_{yi}, of the element to be readily defined. The prime purpose of this definition is to quantify the ultimate curvature, ϕ_{ui}, or

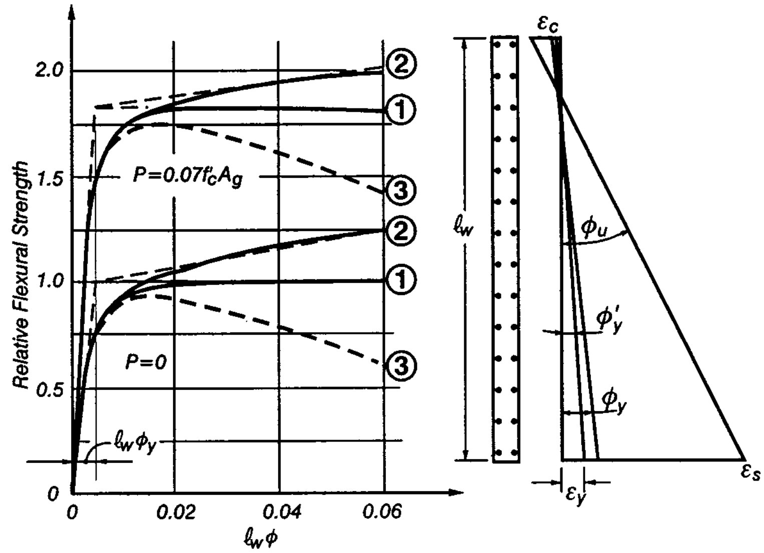

Figure 2. Flexural strength-curvature relationships for a typical reinforced concrete wall section

displacement, Δ_{ui}, as a multiple of the reference yield curvature, ϕ_{yi}, or yield displacement, Δ_{yi}. Based on experimental evidence, this then enables the displacement ductility capacity of an element, which meets the specific requirements of detailing for ductility,[5] to be quantified as

$$\mu_{\Delta\max} = \frac{\Delta_{ui}}{\Delta_{yi}} \tag{11}$$

Because element deflections are conveniently evaluated from the integration of curvatures over the height of the element, particular attention must be given to the reference yield curvature, ϕ_{yi}.

Figure 2 shows typical features of a reinforced concrete rectangular wall section subjected to various levels of curvatures at the base of the cantilever. The nominal flexural strength of the section with a typical quantity of uniformly distributed reinforcement is taken as unity. A moment–curvature analysis indicates that the first yield at the extreme tension fibre occurs when the moment approaches approximately 60 per cent of the nominal strength. A linear extension of the moment–curvature relationship up to the level of the nominal strength may then be taken as the reference yield curvature. The analysis assumes that the concrete does not carry tension stresses. It is indeed appropriate that in seismic design the increased stiffness of uncracked elements be ignored.[7] For design purposes the non-linear relationship, shown in Figure 2, by curves (1), can then be replaced by a bilinear relationship. It is convenient to express curvature in terms of the non-dimensional parameter $l_w\phi$, where l_w is the length of the wall.

Curves (2) in Figure 2 are typical if strain hardening of the tension reinforcement is also taken into account. The corresponding linear approximation, shown by the dashed lines, defines the post-yield stiffness of the element. Curves (3), showing strength degradation, are typically encountered in unreinforced masonry walls. A concentration of additional reinforcement in the boundary region of the wall section does not significantly change the moment–curvature characteristics.

Relationships, plotted in the upper part of Figure 2, show the effects of axial compression due to gravity loads, with an intensity normally encountered in multistorey cantilevers.

Moment–curvature analyses, taking into account the principal parameters of section response, such as material properties, reinforcement content and axial compression load intensity, have shown,[11] that the

reference yield curvature, ϕ_y, does not change significantly in typical rectangular walls with these parameters and that for design purposes the approximation

$$l_w \phi_{yi} = 2\varepsilon_y \tag{12}$$

may be made. ε_y defines the yield strain of the steel used.

If it is assumed that curvatures over the height of a prismatic cantilever vary with the moments that correspond to the pattern of lateral static design forces, the reference yield displacement of the element is given by

$$\Delta_{yi} = \frac{\phi_{yi} h_{wi}^2}{\eta} \approx \left(\frac{2\varepsilon_y h_{wi}^2}{\eta}\right)\frac{1}{l_{wi}} \tag{13}$$

where the coefficient η was defined with equation (2). For example, when for the purpose of a deflection estimate, the conventionally used static design force distribution in the form of an inverted triangle, is replaced by a single concentrated force, applied at the effective height $h_{ei} \approx 0.7\, h_{wi}$, the yield deflection at that level is found when $\eta = 3$.

For a set of wall elements in a typical building all quantities within the brackets of equation (13) will be the same. Therefore, the yield deflection, Δ_{yi}, is inversely proportional to the length, l_{wi}, of the element. When displacement ductilities, $\mu_{\Delta i}$, which are ratios, are addressed, it is sufficient to use relative yield displacements, i.e.,

$$\Delta_{yi} \propto 1/l_{wi} \tag{14}$$

3.2. The relevance of yield displacement to element design

Important features and consequences of the simple relationship expressed by equation (14), not widely appreciated, are:

(i) The yield displacement is *independent* of the strength assigned to the element.

(ii) Elements with different lengths *cannot yield simultaneously*.

(iii) Based on the bilinear lateral force–displacement relationships, similar to those shown in Figure 2, the stiffness of an element with respect to the lateral force, V_{ni}, applied to it, can be defined for design purposes as

$$k_i = \frac{V_{ni}}{\Delta_{yi}} \propto l_{wi} V_{ni} \tag{15}$$

for the elastic range of response. Thus, contrary to the traditional definition by equation (2), stiffness to be considered seismic design is *strength dependent*. It is evident that by increasing the amount of reinforcement in a reinforced concrete structural wall, such as shown in Figure 2, the nominal strength, V_{ni}, will increase. Hence according to equation (15) its stiffness must also increase. This fact is not taken into account in traditional analysis procedures where stiffness is based on the second moment of effective area of the gross concrete section.

(iv) For quantifying the post-yield stiffness, the designer has two options. When material strength enhancement is to be considered, the post-yield stiffness of the element will be

$$k_{pi} = \sigma_i k_i \propto \sigma_i l_{wi} V_{ni} \tag{16}$$

The designer may assume, as the relationship marked (1) in Figure 2 show, that $\sigma_i = 0$. Alternatively from a moment–curvature analysis, allowing particularly for strain hardening of the steel or by making reasonable assumptions, an appropriate value for σ_i may be used. In terms of curvatures, typically $\sigma_i < 0{\cdot}01$, as curves marked (2) in Figure 2 suggest. For poorly detailed elements and particularly for unreinforced brick walls, it will be found that $\sigma_i < 0$, as curves (3) in Figure 2 imply.

(v) With the relative yield displacement, as defined by equation (14), the force, V_{pi}, generated in an element by monotonic displacements can then be estimated by

$$V_{pi} = V_{ni}[1 + \sigma_i(\mu_{\Delta_i} - 1)] \tag{17}$$

The force developed at the attainment of the displacement ductility capacity, $\mu_{\Delta i\max}$, of the element is then

$$V_{i\max} = V_{ni}[1 + \sigma_i(\mu_{\Delta i\max} - 1)] \tag{18}$$

(vi) In the absence of storey twist the lateral displacement capacity of a system is limited by that of its longest element. As a corollary, the displacement ductility capacity of the system may increase in the presence of storey twist in cases when increased displacements due to twist affect only elements with reduced lengths.

As the bilinear approximations in Figure 2 shows, the post-yield section stiffness is about 1–2 per cent of the section stiffness in the elastic range, with the lower values applicable when moderate axial compression is present.[11] When these section stiffness values are translated to member stiffness, relating lateral force to element displacement, post-elastic stiffness are approximately 2·5 times the percentages quoted above. The reason for this is that a displacement ductility demand on a cantilever wall, for example of 5, is associated approximately with a curvature ductility of 12.[7] Therefore, in subsequent examinations of $\sigma = 3$ and 6 per cent will be assumed as typical average and maximum bench mark values of post-yield stiffness when estimating inelastic torsional response.

4. TORSIONALLY UNRESTRAINED SYSTEMS

4.1. A definition of torsional restraint

A typical arrangement of lateral force-resisting elements, such as structural walls, extensively studied in the relevant literature,[10] is shown in Figure 3. With respect to base shear forces, the system is statically determinate. When the nominal strength, V_{ni}, of the elements is such that $V_{n1} = \beta V_{Ey}$ and $V_{n2} = \alpha V_{Ey}$, the two elements should commence yielding simultaneously. The coefficients α and β are defined in Figure 3. However, for the structure, as built, it must be assumed that the element strengths provided will not match exactly the magnitudes dictated by equilibrium criteria. Hence, it must be assumed that when one of the elements, such as (2), commences to yield, the other element, such as (1), will respond within the elastic

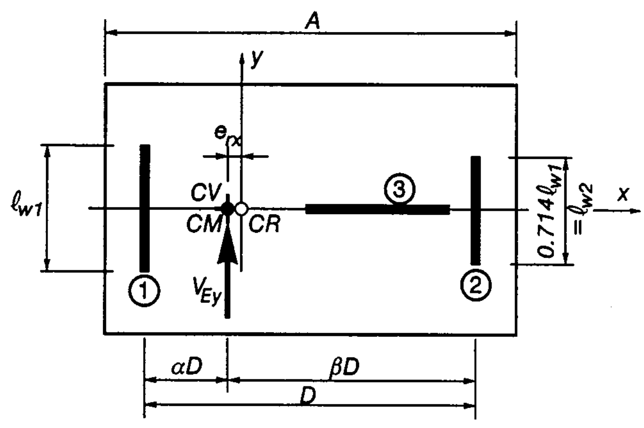

Figure 3. Arrangement of lateral force resisting elements in a torsionally unrestrained system

domain because of its excess strength. If it is assumed that the elements have no post-yield stiffness ($\sigma_i = 0$), displacement ductility should develop only in the yielding element. The displacement profile for such a situation is shown by line (a) in Figure 4(b). Because in the post-yield range of the system there is no resistance against torsion, it is defined as being 'torsionally unrestrained'. The relationship between the displacement ductilities developed in the elements and the system are examined in Section 4.2.

When the post-yield stiffness of elements, k_{pi}, can be relied on, subject to certain conditions, it is possible to develop post-yield displacements of different magnitudes in both elements. Such systems, studied in Section 4.3, may be considered to have limited torsional restraint.

In torsionally unrestrained mechanisms no torsion is generated by static forces in the inelastic range of response, because it cannot be resisted. Torsion can be resisted only within the elastic domain of response. However, this is associated with a corresponding reduction of the base shear capacity, V_{Ey}, of the system. The relationship between the torsional and translatory strength of structural systems has been most effectively demonstrated by illuminating interaction diagrams.[12]

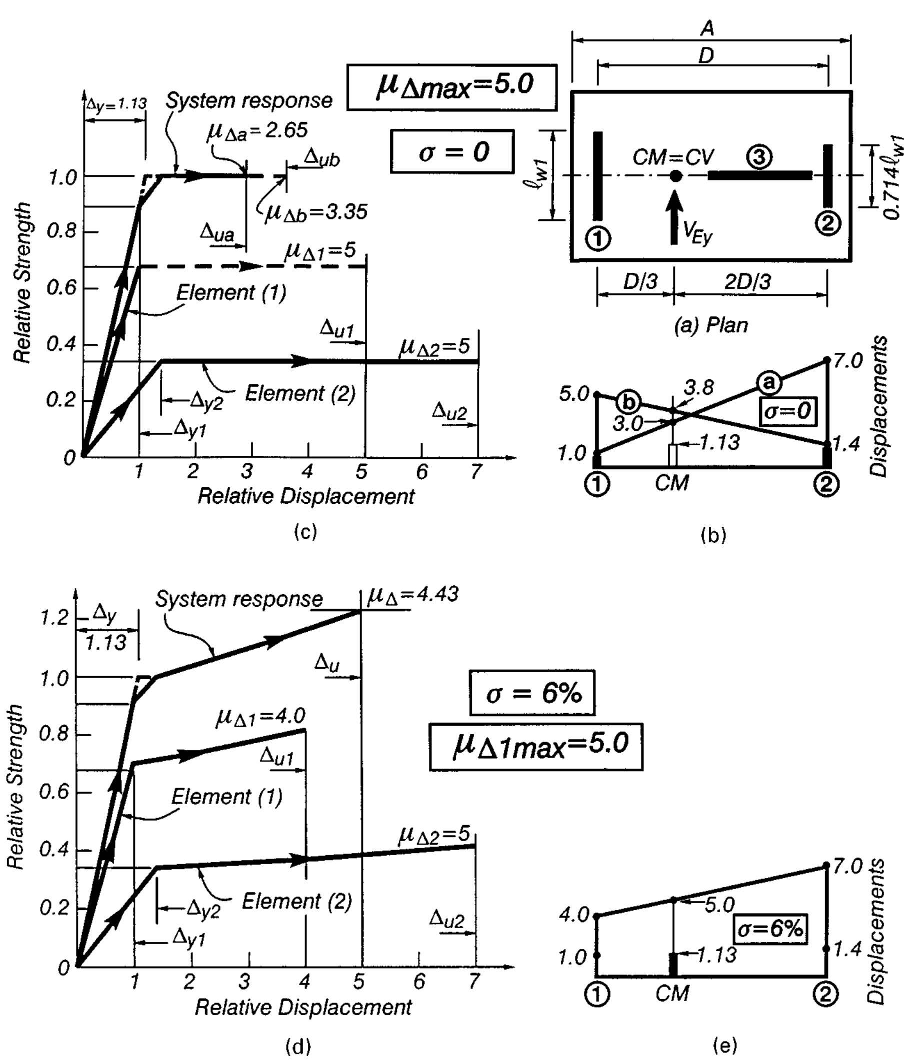

Figure 4. The response of torsionally unrestrained mechanisms with and without post-yield stiffness: (a) plan; (b) displacement patterns; (c) displacements and ductilities; (d) displacements and ductilities; (e) displacement patterns

A comparison of the structural systems shown in Figures 1 and 3 reveals that under the action of a base shear, V_{Ey}, any twisting i.e., rotation of the infinitely rigid diaphragm in its plane, will induce forces also in the transverse elements, labelled (T) in Figure 1. The primary purpose of those elements is, of course, to resist the design base shear in the x direction. It should be appreciated that forces induced at the ultimate limit state, by the base shear in the y direction with a strength eccentricity, e_{vx}, (defined subsequently by equation (27)) as shown in Figure 1, will induce forces in the (T) type elements that are, in general, considerably less than the nominal strength of those elements, controlled by V_{Ex}. Hence, while the force V_{Ey}, is being developed, with few exceptions, those elements can be expected to remain elastic. They will therefore restrain diaphragm rotation and enable the magnitude of the twist, θ_{tu}, shown in Figure 5, to be estimated. These torsionally restrained mechanisms are briefly examined in Section 5. The purpose of the subsequent examinations is to establish a rational relationship between the displacement ductility demand that may be imposed on the structural system and the displacement ductility capacity of elements, particularly those situated at the edges of the floor plan where displacements due to system twist are the largest.[8,9]

4.2. *The response of systems without post-yield stiffness*

If it is assumed that the response of lateral force resisting elements is perfectly elastic–plastic, as shown by the relationships labelled (1) in Figure 2 ($\sigma_i = 0$), it must be assumed that only one of the elements shown in Figure 3 will enter the inelastic domain. To establish the simple relationship, that can be derived as part of the routine design calculations, the example structures shown in Figure 3 is used.

When yielding of element (2) commences at the attainment of its nominal strength, V_{n2}, the force generated in element (1) will be $V_1 = (\beta/\alpha)\, V_{n2}$. When this force is smaller than the nominal strength, V_{n1}, under the action of the static force V_{Ey}, element (1) can never yield unless the resistance of element (2) increases with increased inelastic displacements, i.e., when $\sigma > 0$. The displacement of element (1) at this stage will be

$$\Delta_1 = \frac{V_1}{k_1} = \frac{\beta V_{n2}}{\alpha k_1} \approx \Delta_{y1} \tag{19}$$

If the displacement ductility capacity of element (2), $\mu_{\Delta 2\max}$, is not to be exceeded, the geometry of the displacement pattern, such as shown in Figure 4(b), indicates that the system displacement ductility demand should be limited to

$$\mu_\Delta \leqslant \beta \frac{\Delta_{y1}}{\Delta_y} + \alpha \mu_{\Delta 2\max} \frac{\Delta_{y2}}{\Delta_y} \tag{20}$$

where the system yield displacement, relevant to CM is

$$\Delta_y = \beta \Delta_{y1} + \alpha \Delta_{y2} \tag{21}$$

With the introduction of a geometric system parameter

$$\psi = \frac{\alpha \Delta_{y2}}{\beta \Delta_{y1}} = \frac{\alpha l_{w1}}{\beta l_{w2}} \tag{22}$$

equation (20) simplifies to

$$\mu_\Delta \leqslant \frac{\mu_{\Delta 2\max} + 1}{1 + \psi} \tag{23a}$$

When it is found that in the system shown in Figure 3 it is element (1) rather than element (2) that will yield, it will be found that, to protect element (1) against excessive displacement demands, the system displacement ductility should be limited to

$$\mu_\Delta \leqslant \frac{\mu_{\Delta 1\max} + \psi}{1 + \psi} \tag{23b}$$

In the design of such a torsionally unrestrained structures the system ductility demand, μ_Δ, should be limited to a lesser value, given by equation (23a) or (23b).

To illustrate the significance of these limitations, consider a specific example structure, shown in Figure 4(a), for which the following values are applicable $\mu_{\Delta\max} = 5{\cdot}0$, $l_{w1}/l_{w2} = 1{\cdot}4$, $\alpha = 1/3$ and $\beta = 2/3$ and hence $\psi = 0{\cdot}7$. Therefore from equations (23a) and (23b)

$$\mu_{\Delta a} \leqslant \frac{0{\cdot}7 \times 5 + 1}{1 + 0{\cdot}7} = 2{\cdot}65 \quad \text{or} \quad \mu_{\Delta b} \leqslant \frac{5 + 0{\cdot}7}{1 + 0{\cdot}7} = 3{\cdot}35$$

The critical criterion, $\mu_\Delta \leqslant 2{\cdot}65$, is, as expected associated with a displacement pattern shown by line (a) in Figure 4(b).

The bilinear force–displacement relationships for each of the two elements, as well as for the example system, are shown in Figure 4(c). The full lines are relevant to the case when element (1) does not yield and the dashed lines are applicable when element (2) does not yield.

In Section 2 it was shown how, with the use of typical code specified design stiffness eccentricities, given by equations (9a) and (9b), additional strength is provided in the structure. This traditional approach leads to considerable excess strength to be assigned to one of the two elements. Thereby, this element will never be able to yield unless post-yield stiffness, well above that normally encountered in reinforced concrete elements, is available. An example of a typical case, based on the approximate relative dimensions shown in Figure 3, is presented in Appendix III.

4.3. The response of systems with limited torsional restraint

The examination of the behaviour of the system shown in Figure 4(a) may be extended by considering that the response of the elements exhibits post-yield stiffness, i.e., when $k_{pi} = \sigma k_i$. For typical reinforced concrete elements $\sigma < 0{\cdot}06$.

It is assumed that the post-yield stiffness coefficient, σ, is the same for all elements and that with sufficient precision the yield displacement of the system, Δ_y, can be approximated by equation (21). By considering equilibrium criteria only, the strengths assigned to the two elements are, V_1 and V_2, respectively. To satisfy this requirement the condition $V_1/V_2 = \beta/\alpha$ must be satisfied.

The condition needs now be considered whereby, for example, the nominal strength of element (2) matches the strength assigned to it, i.e. $V_{n2} = V_2$. However, the nominal strength of element (1), V_{n1} is found to be in excess of the strength assigned to it, i.e., $V_{n1} = \lambda_1 V_1$, where $\lambda_1 > 1{\cdot}0$. As the features outlined in Section 4.2 demonstrated, under this condition only element (2) will yield, unless, having post-elastic stiffness, k_{p2}, its strength will sufficiently increase with progressive inelastic displacements. There is an upper limit to the excess strength of element (1), quantified by λ_1, beyond which, for a given post-yield stiffness of element (2), element (1) can never yield. It may be readily shown that under these circumstances post-yield deformation of element (1) can only occur when

$$\lambda_1 < 1 + \sigma(\mu_{\Delta 2\max} - 1) \tag{24}$$

where $\mu_{\Delta 2\max}$ is the maximum ductility demand expected to develop in element (2). Hence for $\mu_{\Delta 2\max} = 5$ typical limits are; $\lambda_{1,\max} = 1{\cdot}12$ when $\sigma = 0{\cdot}03$ and $\lambda_{1,\max} = 1{\cdot}24$ when $\sigma = 0{\cdot}06$.

It may be shown that in this case of limited torsional restraint the system ductility demand should be restricted to

$$\mu_\Delta = \mu_{\Delta 2\max} - \frac{\lambda_1 - 1}{\sigma(1+\psi)} \tag{25}$$

where ψ, a characteristic geometric parameter, is defined by equation (22). Equation (25), the derivation of which is given in Appendix II, is applicable only when $\lambda_1 > 1{\cdot}0$ and $\sigma > 0$.

The structure shown in Figure 4(a) has been reassessed assuming that $\sigma = 6$ percent, a value considered rather large for reinforced concrete elements. The significantly improved displacement pattern is seen in Figure 4(e). The bilinear force–displacement relationship for the two elements and the system are presented in Figure 4(d). The two-fold benefits of the large post-yield stiffness are in this case; an increase of the system ductility capacity from $\mu_\Delta = 2{\cdot}65$ to $\mu_\Delta = 4{\cdot}43$ and a strength increase by approximately 25 per cent at the development of this system displacement ductility capacity.

4.4. Implications for design

The identification of plastic mechanisms of structural systems which, in terms of storey twist, are partially, or not restrained suggests the following design considerations:

1. When it is appropriate to make the conservative assumption that translatory elements, such as shown in Figures 3 and 4(a), do not posses post-yield stiffness, the system displacement ductility demand, μ_Δ, with respect to CM should be limited by equations (23a) and (23b).
2. When post-yield stiffness of elements, defined by the stiffness coefficient σ, can be relied on, the system displacement ductility, μ_Δ, should be limited in accordance equation (25).
3. When using established force-based design procedures, the design base shear should be that corresponding to specified elastic spectra, reduced in recognition of the system displacement ductility capacity, μ_Δ, as defined in paragraphs (1) and (2) above.
4. When a displacement based seismic design strategy is used, the displacement capacity of the system, measured at CM, is defined as $\Delta_u = \mu_\Delta \Delta_y$, where the absolute yield displacement can be evaluated with equation (21). The absolute yield displacement of elements, Δ_{yi}, is given by equation (13).
5. In situations, such as shown in Figure 4(b), where the excess strength of one of the elements results in greater restriction on the acceptable system displacement ductility demand, μ_Δ, the designer may choose to allow only the mechanism associated with more favourable system ductility to be mobilized. With the use of the principles of capacity design this may be readily achieved. For example in the structure shown in Figure 4(a), the designer may inhibit the yielding of element (2). This may be achieved if its nominal strength, V_{n2}, is more than that, V_2, which corresponds with the overstrength of the ductile element (1), i.e.

$$V_{n2} > V_2 = (\alpha/\beta)[1 + \sigma(\mu_{\Delta\max} - 1)]\, V_{n1} \tag{26}$$

   For this case a reasonably large post-yield stiffness coefficient, such as $\sigma = 0{\cdot}06$, should be assumed. The advantage of this strategy of utilising only one of the two possible torsional mechanisms is that element (2), remaining elastic, need not be detailed for ductile response.

5. TORSIONALLY RESTRAINED SYSTEMS

In torsionally restraint mechanisms earthquake-induced torque can be resisted at the ultimate limit state even when all translatory elements in one of the principal directions respond in the plastic range.

As explained in Section 4.1, this restraint is provided by transverse elements located in at least two planes, which respond in the elastic domain. Therefore, they control the angle of twist. An example structure,

shown in Figure 1, will be used to illustrate briefly the simple salient features of the relevant torsional mechanisms. Design recommendations, not recapitulated here, have been proposed for such desirable systems.[8, 9]

5.1. The centre of resistance

Based on traditional approaches or using other concepts,[8] implied in Section 3.2, which offer considerable freedom of choice, the designer allocates strength to each of the translatory elements, such as (1)–(4) in figure 1. Based on nominal element strengths, V_{ni}, that have been provided, the location of the centre of resistance of these inelastic translatory elements, denoted as CV, may be readily found. The distance between CM and CV is then the strength eccentricity, defined with respect to elements (1)–(4) in Figure 1 by

$$e_{vx} = \frac{\sum x_i V_{ni}}{\sum V_{ni}} \tag{27}$$

where x_i is the distance of the element from CM. The condition of equation (27) being valid is that the possibility of developing the nominal strength of all translatory elements must exist. When one or more elements cannot achieve their nominal strength, V_{ni}, than the computed reduced resistance of such elements must be used in equation (27). This situation will commonly arise in torsionally unrestrained systems. It may also arise in torsionally restrained structures in which the transverse elements, providing torsional resistance, yield before the nominal strength of all translatory elements is developed. These conditions are controlled by equilbrium criteria only. A situation to which the restriction is relevant is described at the end of Section 6.1. It is emphasized that, provided that the condition $\Sigma V_{ni} \geqslant V_E$ is satisfied, the assignment of the nominal strength of translatory elements is the designer's choice. A perceptive choice will lead to an advantageous location of CV and hence will promote optimal system response.

5.2. The prediction of the angle of twist

The resulting torque, $e_{vx}\Sigma V_{ni}$, is resisted by the elastic transverse (T) type elements (Figure 1). The angle of twist at the ultimate limit state, θ_{tu}, is obtained by equation (6) in which the torsional stiffness accounts only for the contributions of the elastic transverse elements. In the example considered, only the second term of equation (3), referred to as the 'residual torsional stiffness',[13] is relevant. Therefore,

$$\theta_{tu} = \frac{e_{vx} \sum V_{ni}}{\sum y_{ri}^2 k_{xi}} \tag{28}$$

where the stiffness of the (T) type elements, k_{xi}, is defined by equation (15). It should be noted that the stiffness of these elements depends on the strength assigned to them as a result of the design base shear, V_{Ex}. In the example structure in Figure 1, it was assumed that the four (T) elements are identical.

5.3. The estimation of variable inelastic element displacement demands

Once the storey twist at the ultimate limit state, based on infinitely rigid diaphragm behaviour, has been established from equation (28), the linear variation of the ultimate displacements of the ductile translatory elements, including CM, is known. Such a displacement pattern for the structure shown in Figure 1 is seen in Figure 5. The yield displacement, Δ_{yi}, for each element is also shown. As demonstrated in Section 3.1, these are inversely proportional to element lengths, l_{wi}.

An inspection of figure 5 and a comparison of the element ultimate, Δ_{ui}, and yield displacements, Δ_{yi}, indicates which element is subjected to the largest displacement ductility demand. If this is to be limited to the displacement ductility capacity, $\mu_{\Delta i\max}$, of the elements, the ultimate displacement demand on the system, Δ_u, at the centre of mass is readily found from the geometry of the displacement pattern.

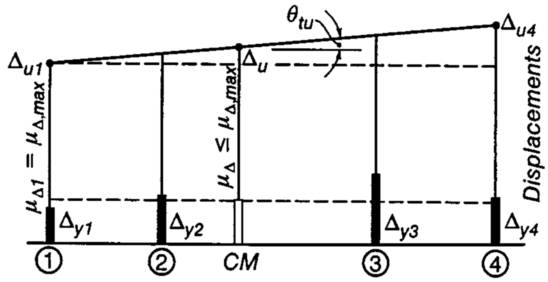

Figure 5. Displacement pattern for a torsionally restrained system

The system yield displacement, Δ_y, will be in this case some mean value of the element yield displacements. It has been suggested[8] that, based on bilinear force–displacement simulation, for torsionally restrained systems

$$\Delta_y = \frac{\sum V_{ni}}{\sum k_i} \tag{29}$$

is a satisfactory approximation. Element stiffness, k_i, is defined by equation (15). Therefore, the limitation on the system displacement ductility demand, $\mu_\Delta = \Delta_u/\Delta_y$, is established, details of which were given elsewhere.[8] Note that the definition of the yield displacement by equation (21) for torsionally unrestrained systems is slightly different.

5.4. The main features of torsionally restrained mechanisms

Elastic transverse elements will restrict the angle of twist, θ_{tu}, which results from the storey shear force being applied with a strength eccentricity, e_v.

When elements of different lengths are used, a strength eccentricity, e_v, sometimes quite significant, may be beneficial by increasing the acceptable system displacement ductility demand. Figure 5 shows for example, that the torque and hence the angle of twist could be increased by some 50 per cent before the displacement ductility capacity of element (4) at the opposite edge of the plan would be reached. As a corollary, no strength eccentricity, resulting in identical translation of all elements, would significantly reduce the acceptable system displacement ductility demand.

For a given system, such as shown in Figure 1, the designer has considerable freedom in assigning strength to elements, whereby, if desired, an optimum magnitude of the strength eccentricity can be achieved without necessarily changing element dimensions. A strategy for strength allocation to elements has also been proposed.[8,9]

The stiffness of the elastic transverse elements, such as the type (T) elements in Figure 1, will depend on the ductility demand imposed on them during preceding inelastic displacement excursions in the transverse direction of the plan (V_{Ex}), when they acted as ductile translatory elements. The designer may wish to check the consequences of stiffness degradation of these elements when, according to equation (28) they are expected to provide torsional stiffness.

6. ASSESSMENT OF EARTHQUAKE-IMPOSED SKEW DISPLACEMENT DUCTILITY DEMANDS

When large earthquake-induced diagonal displacements are imposed on the structural system, all elements of a structure, such as shown in Figure 1, in both directions, must be expected to enter the inelastic range of response. Assuming that the elements have no post-yield stiffness ($\sigma = 0$), they will then not be able to resist

any additional forces that may result from torsion. Therefore, under seismic displacement in the y direction, torsional restraint offered previously by elastic transverse elements, such as the type (T) elements, will no longer be available. At this instant the translatory mechanism in the y direction degenerates into a torsionally unrestrained one. These mechanisms were examined in Section 4 and were shown in Figures 3 and 4.

For the evaluation of the torsional mechanisms which could develop during significant inelastic skew displacements of the system, the simultaneous orthogonal formation of torsionally unrestrained mechanisms should be considered.

6.1. A comparison of torsionally restrained and unrestrained mechanisms

To aid a better understanding of the inelastic response of systems subjected to skew earthquake attacks, a comparison of two distinct mechanisms is made. This review will also serve as a summary of the characteristic features of each of the two systems.

Figure 6(a) shows an idealized torsionally restrained mechanism comprising only four elements. The heavily outlined plan shows the rigid body deformation of the diaphragm relative to its original position before seismic displacements occurred. With a base shear V_{Ey} the ultimate system displacement, measured at CM, is Δ_{uy}. Because of system twist the yielding elements, (1) and (3), shown as solid blocks, are subjected to different displacements. The twist, θ_{tuy}, is a consequence of the torque generated by V_{Ey} when acting with a strength eccentricity, e_{vx}. This torque is resisted entirely by the elastic transverse elements (2) and (4), which thus control the angle of twist, θ_{tuy}, at the ultimate limit state. Figure 6(b) shows a similar mechanism corresponding to the base shear, V_{Ex}, in the x direction. Figures 6(a) and 6(b) depict conditions encountered in routine design practice.

Should elements (2) and (4) be replaced by a single element resisting V_{Ex}, such as seen in Figure 3, for the sake of clarity not shown in Figures 6(c) and (d), a torsionally unrestrained mechanism ensues. It cannot sustain any torque! Should the nominal strength of elements (1) exceed the demand controlled by both equilibrium and the strength of the yielding element (3), element (1) will not yield unless elements have post-yield stiffness. Hence the ultimate system displacement, Δ'_{uy}, will be associated with a significantly larger angle of twist, θ'_{tuy}, than in the case shown in Figure 6(a).

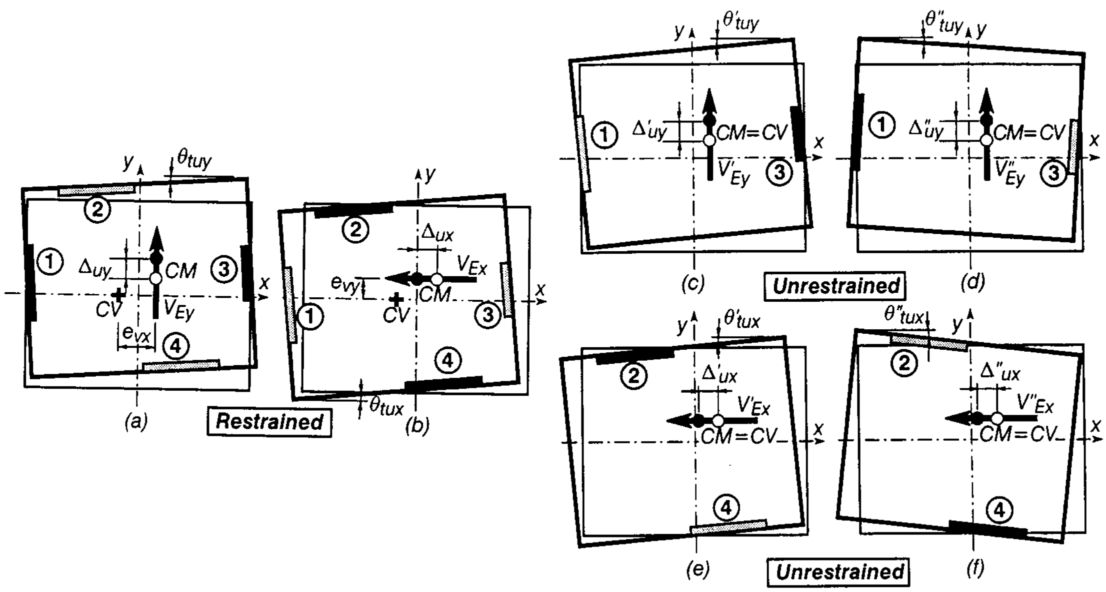

Figure 6. A comparison of torsionally restrained and unrestrained mechanisms

When the strength of element (3) is in excess of the maximum possible demand associated with the yielding of element (1), the situation depicted in Figure 6(d) will arise. The strength relationship between elements (1) and (3) of a real structure, as detailed or constructed, can be readily determined. Hence the relevant mechanisms and angles of twist, θ'_{tuy}, or θ''_{tuy}, associated with the development of the displacement ductility capacity of the yielding element, rather than any torsional restrained, as shown in Figures 6(c) and (d), are defined. Note that in these cases a strength eccentricity $e_{\mathrm{v}x}$, cannot exist, i.e., CM = CV.

Figure 6(e) and (f) show similar torsionally unrestrained mechanisms associated with a base shear in the x direction, $V_{\mathrm{E}x}$, which would result if elements (1) and (3) were to be replaced with a single element resisting $V_{\mathrm{E}y}$.

6.2. Torsionally unrestrained systems developing under skew displacements imposed on torsionally restrained systems

Figure 7 shows stages in the development of possible torsional mechanisms due to earthquake-induced skew displacements imposed on the systems, shown in figures 6(a) and (b), which were torsionally restrained under attack along either of the two principal orthogonal directions of the plan. The concept of the superposition of two mechanisms is utilised. This is admissible but only because each mechanism engages different elements.

First, it is assumed that as a result of the x component of the imposed diagonal inelastic displacement, one or both elements in this direction, i.e., elements (2) and (4), entered the inelastic range of response. Hence this pair of elements is unable to resist any torsion. The response of elements (1) and (3), shown in Figure 7(a) need now be studied. As at this stage there can be no strength eccentricity ($e_{\mathrm{v}x} = 0$), the magnitudes of the force in element (1) and of the total base shear $V^*_{\mathrm{E}y}$ are set by equilibrium requirement only when element (3) develops its nominal strength. The role of the post-yield stiffness coefficient may be, but is presently not considered, i.e., $\sigma = 0$. It is assumed that, as part of the routine design, the strength of elements (1) and (3) has been based on the model shown in Figure 6(a). Therefore, under the conditions depicted in Figure 7(a), the force generated in

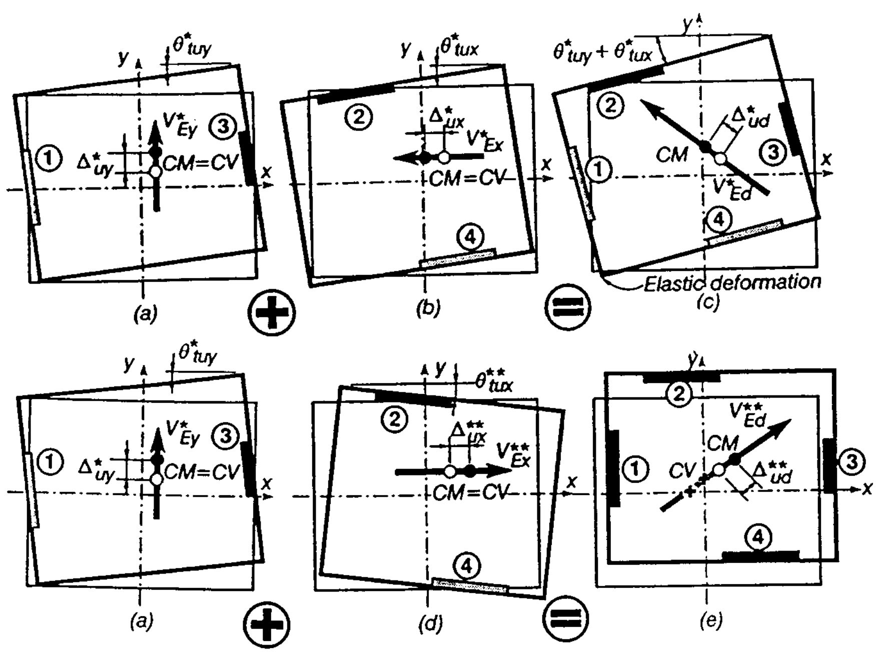

Figure 7. The decomposition of torsional mechanisms with skew seismic attack

element (1) will be less than its strength, V_{n1}. The position of CV in Figure 6(a) verifies this. Consequently in the mechanism shown in Figure 7(a) element (1) will remain elastic.

The subtle difference between element strengths of the torsionally unrestrained systems shown in Figures 6(c) and (d) and that in Figure 7(a) should be noted. In the first, the location of the elastic elements it is not obvious. It depends on the relative strengths of the elements as provided in the construction. In the latter case, however, the identification of the element which remains elastic is definitive, i.e., element (1) as shown in Figure 7(a).

An important feature of the conditions shown in figure 7(a) is that, on account of element (1) not being able to develop its nominal strength, the lateral base shear $V^*_{Ey} = V_1 + V_{n3}$ is less than V_{Ey}, the design base shear to be developed in the y direction only, shown in Figure 6(a).

The torsionally unrestrained mechanism shown in Figure 7(b) is similarly based on the necessary assumption that under imposed inelastic deformations on any of the elements in the y directions, those elements have been rendered ineffective to resist torsion. A comparison of Figures 7(b) and 6(e) shows that for the given direction of the base shear, V^*_{Ex}, it is now element (4) which remains elastic, and that

$$V^*_{Ex} = V_{n2} + V_4 < V_{Ex} \tag{30}$$

Finally, the superposition of the mechanisms in Figures 7(a) and 7(b) results in the mechanisms shown in Figure 7(c). It shows that:

(i) The angles of twist are additive possibly resulting in large total twist.
(ii) Elements (1) and (4) remain elastic and hence to total displacement at the lower left-hand corner of the plan is relatively small.
(iii) The total acceptable angle of twist is determined by the ultimate displacement ductility capacity, $\mu_{\Delta i\max}$, which could be developed in the ductile elements (2) and (3). This should be checked.
(iv) The designer is not expected to attempt to predict the magnitude of the total ultimate skew displacement of the system, Δ^*_{ud}. However, with the knowledge of the displacement ductility capacities of the two critical elements, i.e., (2) and (3), the magnitude of the maximum acceptable skew mass displacement can be readily estimated from

$$\Delta^*_{ud} = \sqrt{(\Delta^*_{ux})^2 + (\Delta^*_{uy})^2} \tag{31}$$

As a conservative approximation the designer may assume that the centre of torsional rotation is at the intersection of the planes of the elastic elements (1) and (4). Hence by considering the maximum acceptable displacements in elements (2) and (3), the angle of twist and hence Δ^*_{ud} may be estimated.
(v) By ignoring the contribution of post-yield stiffness of the base shear that could, but need not, develop in this critical diagonal direction is

$$V^*_{Ed} = \sqrt{(V^*_{Ey})^2 + (V^*_{Ex})^2} \tag{32}$$

6.3. The critical direction of seismic attack

The case considered previously, as shown in Figures 7(a)–7(c), is considered to produce the most adverse conditions. This is because both components of the imposed diagonal displacement produce twist in the same sense.

When the case shown in Figure 7(a) is combined with that in Figure 7(d), resulting in twists with opposite senses, a condition shown in Figure 7(e) may result. In the optimum situation the displacement induced diagonal force vector, V^{**}_{Ed}, would pass through both the original centre of resistance, CV and CM. Hence no strength eccentricity would exist and all four of the ductile elements could develop their full strength. Hence,

from figures 6(a) and 6(b) the resulting force may reach its maximum value of

$$V^{**}_{\mathrm{Ed}} = \sqrt{(V_{\mathrm{E}y})^2 + (V_{\mathrm{E}x})^2} > V^{*}_{\mathrm{Ed}} \tag{33}$$

The displacement ductility demand on any of the elements is thus likely to be less than that arising during imposed displacements separately in the y and x directions. Therefore, this case should not need to be addressed by the designer.

6.4. *Implications of skew seismic attacks*

The purpose of the identification of torsional mechanisms that could be mobilized by earthquake-imposed inelastic skew displacements of the centre of mass, is to enable comparisons to be made with estimated responses with respect to displacements applied separately in the principal directions of the building. Such a comparison may be based on the notion that the ultimate displacement and base shear capacities of the system with respect to any orientation of the seismic motions should be of the same order. The evaluation of the maximum acceptable skew displacement in accordance with equation (31) and the associated base shear, given be equations (32) or (33), enables such comparison to be readily made. In structural systems with distinctly different periods of vibration with respect to the principal axes of the plan, an elliptic boundary may be assumed for the estimation of the ultimate displacement in any direction.

Whenever it is found that

$$\Delta_{\mathrm{u}y} < \Delta_{\mathrm{ud}} > \Delta_{\mathrm{u}x} \tag{34}$$

and

$$V_{\mathrm{E}y} < V_{\mathrm{Ed}} > V_{\mathrm{E}x} \tag{35}$$

where the ultimate displacements and base shear forces are identified in figures 6(a), and (b), and Figures 7(c) and 7(e), respectively, features of skew seismic attacks should not cause concern.

7. CONCLUSIONS

1. A rational approach to the seismic design of building structures, which are expected to respond in a ductile manner, requires that uniquely definable plastic mechanisms can be mobilized. This principle is widely recognized and applied when the translatory response of frames or walls is considered. However, no equivalent approach to the definition of mechanisms involving torsion, which is addressed in this paper, appears to have been formulated.
2. When torsion-induced displacements occur, the primary aim of the design strategy should be to ensure that the expected displacement demand on the system does not lead to demands that exceed the displacement ductility capacity of elements, particularly those remote from the centre of twist. The likely critical elements are predominantly those which have the smallest yield displacement.
3. The identification and understanding of plastic mechanisms, as affected by system twist, enables the acceptable system displacement ductility demand, as a function of the ductility capacity of the critically situated vertical lateral force resisting element, to be estimated.
4. Three-dimensional mechanisms must be kinematically admissible while satisfying simple equilibrium criteria.
5. Two fundamentally different mechanisms are postulated. In one, much to be preferred, elastic transverse elements are assigned to resist torsion and hence to control system twist, while translatory elements are subjected to inelastic displacements of different magnitudes. This mechanism is identified as being 'torsionally restrained'. In the other system, preferably to be avoided, elements capable of resisting torsion during inelastic translatory response, are absent. As a consequence one edge element

may not enter the inelastic domain while the element at the opposite edge of the plan may be subjected to excessive ductility demands. This mechanism is defined as being 'torsionally unrestrained'.

6. The identification of mechanisms which can be mobilised separately in each of the two principal orthogonal directions of the building, enables also the maximum potential inelastic displacement and base shear capacity, associated with skew earthquake attacks, to be estimated. These limits depend also on the predetermined displacement ductility capacity of readily identifiable critical elements. Torsionally restrained mechanisms subjected to inelastic skew displacements must be expected to degenerate into a torsionally unrestrained mechanism.
7. It is claimed[8,14] that relevant code provisions, used the world over and based on the properties of elastic systems, do not address the vital feature of inelastic seismic response, the maximum element displacements generated. The key parameters of these code provisions are adjustable stiffness eccentricities, utilised to provide increased torsional resistance. The important quantity relevant to ductile response is claimed to be strength rather than stiffness eccentricity, which realistically gauges the torque to be sustained at the ultimate limit state.
8. The magnitude of the strength eccentricity is under the control of the designer. Without changing acceptable element dimensions, strength to translatory elements can be assigned or redistributed so as to result in a favourable strength eccentricity. A reduction or elimination of strength eccentricity in systems comprising elements with different yield displacements, does not necessarily result in a more even distribution of element displacement ductility demands.
9. An identification of torsional mechanisms may be of considerable benefit at the conceptual stage of structural design. Moreover, it enables also the potential displacement capacity of existing systems, comprising elements with identified strength properties and displacement ductility capacities and requiring a seismic review, to be estimated. Established code provisions for torsional effects cannot be utilized for this purpose.
10. The postulates enumerated also offer a challenge to researchers to investigate, by means of analyses and realistic modelling of non-linear element response, the differences between the dynamic system response to selected earthquake records and the simple estimates based on monotonically imposed displacements on selected or identified torsional mechanisms. Some relevant studies are currently being undertaken.

APPENDIX I

NOTATION

The following symbols are used in this paper

CM	centre of mass
CR	centre of rigidity or centre of stiffness of shear centre of the structural system
CV	centre of resistance or centre of strength at the ultimate limit state
e_{d1}, e_{d2}	design stiffness eccentricities
e_r, e_{rx}, e_{ry}	stiffness eccentricities
e_{vx}, e_{vy}	strength eccentricities
E_c	modulus of elasticity of concrete
f'_c	congression strength of the concrete, MPa
h_{ei}	effective height of a wall
h_{wi}	height of structural wall
I_e	effective second moment of area of a section
k_i	stiffness of element relating lateral force to lateral displacement at a given level

k_{pi}	post-yield stiffness of element
k_{xi}, k_{yi}	stiffness of element with respect to x and y directions, respectively
K_{sy}	translatory stiffness of the structural system
K_t	torsional stiffness of the structural system
l_w, l_{wi}	length of structural wall section
r_k, r_{kx}, r_{ky}	radius of gyration of element stiffness
V_{Ed}	base shear developed due to skew displacements
V_{Ex}, V_{Ey}	design base shear assigned to the system with respect to the x or y directions
V^*_{Ex}, V^*_{Ey}	reduced base shear capacity of the system
V_i	base shear generated in an elastic element
V''_i	element force induced by torsion only
V_{Ei}	design base shear assigned to an element
V_{ni}	nominal base shear strength of an element
V_{pi}	base shear developed when the displacement ductility demand imposed on element i is larger than unity but smaller than its ductility capacity
x_i, y_i	distance of element from CM
x_{ri}, y_{ri}	coordinates of element with respect to CR
Δ_{Ey}	translational displacement of an elastic system
Δ_i	displacement of an element
Δ_u	ultimate displacement of the structural system
Δ_{ud}	ultimate skew displacement of the system
Δ_{ui}	ultimate displacement imposed on an element
Δ_{ux}, Δ_{uy}	ultimate system displacements in the x and y directions, respectively
Δ_y	yield displacement of the structural system
Δ_{yi}	yield displacement of an element
ε_y	yield strain of the steel
θ_t	angle of twist
θ_{tu}, θ_{tuy}	angle of twist at the ultimate limit state
λ_I	a strength coefficient
μ_Δ	displacement ductility capacity of the structural system with respect to displacements at CM
$\mu_{\Delta i}$	displacement ductility demand imposed on element
$\mu_{\Delta \mathrm{imax}}$	displacement ductility capacity of element
σ, σ_i	ratio of post-yield to pre-yield stiffness of element
ϕ_{ui}	curvature at the ultimate limit state
ϕ_{yi}	equivalent yield curvature of the element
ψ	a geometric system parameter

APPENDIX II

The determination of the system displacement ductility capacity when limited torsional restraint is available.

The derivation is specifically relevant to the system described in Section 4.3 and shown in Figure 3. All variables are defined in the list of notations.

1. The forces induced in the two elements by a unit base shear, V_{Ey}, acting at CM, are such that $V_{E1}/V_{E2} = \beta/\alpha$ and $V_{E1} + V_{E2} = 1{\cdot}0$.
2. The nominal strengths of the elements, as constructed, are such that $V_{n2} = V_{E2}$ and $V_{n1} = \lambda_1 V_{E1}$ where $\lambda_1 > 1{\cdot}0$.
3. The limitation of equation (24) applies, i.e., $\lambda_1 < 1 + \sigma(\mu_{\Delta 2\max} - 1)$.

4. At the onset of yielding of element (1) the force developed in element (2) is $V^*_{p2} = (\alpha/\beta) V_{u1} = (\alpha/\beta)\lambda_1 V_{E1} < V_{p2}$, where the maximum force developed in element (2), if subjected to its full displacement ductility capacity, is $V_{p2} = [1 + \sigma(\mu_{\Delta 2\max} - 1)] V_{n2}$.
5. Inelastic displacement in element (1) will occur as a result of the force increment in element (2), i.e., $\Delta V_{p2} = V_{p2} - V^*_{p2}$ resulting in a corresponding force increment in element (1) of $\Delta V_{p1} = (\beta/\alpha)(V_{p2} - V^*_{p2})$. When substituting from steps (1) and (4), this increment become $\Delta V_{p1} = [\sigma(\mu_{\Delta 2\max} - 1) - \lambda_1 + 1] V_{E1}$.
6. The resulting inelastic displacement of element (1) is thus

$$\Delta_{p1} = \frac{\Delta V_{p1}}{k_{p1}} = \left[\mu_{\Delta\max} - 1 - \left(\frac{\lambda_1 - 1}{\sigma}\right)\right]\Delta_{y1} \quad \text{where} \quad \Delta_{y1} = \frac{V_{E1}}{k_1}$$

7. The ultimate displacements of the two elements, when element (2) attained its displacement ductility capacity, are thus $\Delta_{u1} = \Delta_{y1} + \Delta_{p1} = [\mu_{\Delta 2\max} - (\lambda_1 - 1)/\sigma]\,\Delta_{y1}$ and $\Delta_{u2} = \mu_{\Delta 2\max}\Delta_{y2}$
8. The ultimate displacement of the system at CM is $\Delta_u = \beta\Delta_{u1} + \alpha\Delta_{u2}$. When substituting from step (7), it is found that $\Delta_u = \mu_{\Delta 2\max}\Delta_y - [(\lambda_1 - 1)\beta\Delta_{y1}]/\sigma$, where the system yield displacement, Δ_y, is defined by equation (21).
9. The system displacement ductility demand should therefore be limited to

$$\mu_\Delta = \frac{\Delta_u}{\Delta_y} = \mu_{\Delta 2\max} - \frac{(\lambda_1 - 1)\beta\Delta_{y1}}{\sigma\Delta_y}.$$

The expression $\beta\Delta_{y1}/\Delta_y$ simplifies to $1/(1+\psi)$, where according to equation (22) $\psi = \alpha l_{w1}/\beta l_{w2}$. Therefore, $\mu_\Delta = \mu_{\Delta 2\max} - (\lambda_1 - 1)/\sigma(1+\psi)$ [equation (25)].

APPENDIX III

A torsionally unrestrained system complying with existing code requirements

In accordance with typical code requirements the accidental design eccentricity associated with the lateral static design forces is taken as $\pm 0{\cdot}1$ A. To study its effect on the example structure shown in Figure 3(a) is considered. Accordingly, the design forces to be assigned to the two elements with identical thicknesses, shown in Figure 3 and based on the following properties, are as follows:

$$I_1 \propto l_{w1}^3 = 1{\cdot}0, \quad I_2 \propto (0{\cdot}714 l_{w1})^3 = 0{\cdot}364, \quad \Sigma I_i = 1{\cdot}364, \quad \sigma = 0$$

$$e_{rx} = (-1{\cdot}0\alpha + 0{\cdot}364\beta)D/1{\cdot}364 = 0{\cdot}007D \text{ when } \alpha = 0{\cdot}26 \text{ and } \beta = 0{\cdot}74.$$

From equations (9a) and (9b) and $0{\cdot}1A = 0{\cdot}125D$

$$e_{d1} = (0{\cdot}007 - 0{\cdot}125)\mathrm{D} = -0{\cdot}118\mathrm{D} \text{ and } e_{d2} = (0{\cdot}009 + 0{\cdot}125)\mathrm{D} = 0{\cdot}132\mathrm{D}$$

$$\text{Hence, } V_{n1} \geqslant (1/1{\cdot}364 + 0{\cdot}118)V_{Ey} = 0{\cdot}851\ V_{Ey} \text{ and } V_{n2} \geqslant (0{\cdot}364/1{\cdot}364 + 0{\cdot}132)V_{Ey} = 0{\cdot}399\ V_{Ey}$$

Hence $\Sigma V_{ui} = 1{\cdot}25\ V_{Ey}$, what is often referred to as the overstrength.[10]

For the two elements, with no post-yield stiffness, to yield simultaneously, the ratio of the strength at element (1) to that of element (2) must be exactly $\beta/\alpha = 2{\cdot}846$. However, the code required design strength ratio is $V_{n1}/V_{n2} = 0{\cdot}851/0{\cdot}399 = 2{\cdot}133 < 2{\cdot}846$.

Therefore, element (2) will never yield unless, due to its post-yield stiffness, the strength of element (1) increases to a level, V^*_{n1}, which could be achieved within the typical displacement ductility capacity, $\mu_{\Delta\max} \approx 5$, of reinforced concrete walls with a post-yield stiffness encountered in practice. i.e., $V^*_{n1} = 2{\cdot}846 \times 0{\cdot}399\ V_{Ey} = 1{\cdot}14\,V_{Ey} = 1{\cdot}34\ V_{u1}$. Using equation (4) it is found that the value of σ would need to be in excess of 8·5 per cent to develop any yielding in element (2).

From equation (22) $\psi = (0{\cdot}26 \times 1{\cdot}00)/(0{\cdot}76 \times 0{\cdot}71) = 0{\cdot}46$. Hence from equation (23b) $\mu_\Delta = (5 + 0{\cdot}46)/(1{\cdot}0 + 0{\cdot}46) = 3{\cdot}74 < 5{\cdot}0$.

This finding suggests that existing code provisions would aggravate the response of torsionally unrestrained mechanisms by ensuring that only one element will yield. Code provision simply increased in this example the lateral strength of the system by 25 per cent without addressing the important issues relevant to displacement ductility.

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